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Ratio Estimator of Population Mean in Simple Random Sampling

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Abstract: This paper considers the problem of estimating the population mean in Simple Random Sampling. One key objective of any statistical estimation process is to find estimates of parameter of interest with more efficiency. It is well established that incorporating additional information in the estimation procedure gives enhanced estimators. Ratio estimation improves accuracy of the estimate of the population mean by incorporating prior information of a supporting variable that is highly associated with the main variable. This paper incorporates non-conventional measure (Tri-mean) with quartile deviation as they are not affected by outliers together with kurtosis coefficients and information on the sample size to develop an estimator with more precision. Using Taylor series expansion, the properties of the estimator are evaluated to first degree order. Further, the estimator's properties are assessed by bias and mean squared error. Efficiency conditions are derived theoretically whereby the suggested estimator performs better than the prevailing estimators. To support the theoretical results, simulation and numerical studies are undertaken to assess efficiency of the suggested estimator over the existing estimators. Empirical analysis done through percentage relative efficiency indicate the suggested estimator performs better compared to the prevailing estimators. It is concluded that the suggested estimator is more efficient than the existing estimators.

Keywords: Ratio Estimator, Non-conventional Location Parameters, Auxiliary Information, Mean Squared Error

1. Introduction

In statistical estimation, the parameter of interest is estimated with the characteristics of unbiasedness, consistency and efficiency. The mean per unit estimator of the study variable is a suitable estimator which is considered fit for estimating the population mean. It is unbiased but also has a lot of variance which is undesirable. Of importance therefore is to get estimates of parameter of interest with better accuracy and least mean squared error. Therefore, we integrate more information into the estimation process to produce better estimators. Ratio estimation utilizes auxiliary information on a variable being highly positively correlated with the main variable so as to attain an estimate of the population mean. Additionally, this form of estimation is most efficient when the auxiliary and study variables have a linear association as well as are positively correlated.

Cochran [2] pioneered utilization of auxiliary information in developing a ratio estimator for the population mean. In the event that the main and supporting variables are positively correlated, the ratio type estimator is a better estimator than the simple mean estimator as it is more efficient while Robson's [3] product estimator is more efficient compared to the simple mean estimator if the correlation among the two variables is negative. Further enhancements to the classical ratio estimator are also achieved by use of known population characteristics that include the skewness and kurtosis coefficients, variation coefficient and correlation coefficient. Srivenkataramana and Tracy [8], Upadhyaya and Singh [11], Singh and Tailor [7], Kadilar and Cingi [5], Yan and Tian [14], Subramani and Kumarapandiya [9], Jeelani, et al., [4], Shittu and Adepoju [6], Abid, et al., [1] may be referred to for more detailed discussion.

Further, Subzar, et al. [10] constructed ratio estimators by use of non-conventional position parameters which include mid-Range and tri-Mean, Hodges-Lehmann with skewness and kurtosis coefficients. Yadav, et al., [13] used both conventional and non-conventional measures that include quartile deviation, decile mean, tri-mean, mid-range, Hodges-Lehmann, Downton's method, Probability weighted moments, Gini's Mean Difference as auxiliary information together with information on the sample size to develop ratio estimators under simple random sampling.

In this paper we suggest an improved ratio type estimator by use of quartile deviation, tri-mean, coefficient of kurtosis and information on the size of the sample. Consider a finite population H (H_1, H_2, …, H_N) of N different as well as distinguishable units. Consider to Y be the main variable with Y_i taken on H_i, i = 1, 2, ….., N. The objective to get an estimate for the population mean.

Subzar, et al. [10] presented a class of ratio estimators by use of both traditional measures and non-traditional measures like Tri-Mean, Mid-Range and Hodges-Lehmann as auxiliary information. These estimators are given as:

$$t_{nb} = \frac{\bar{y} + \mathrm{b}(\bar{X} - \bar{x})}{(\bar{x} + \alpha_j)} \left(\bar{X} + \alpha_j\right), b = 1,2, \dots 6, j = 1,2, \dots, 6 \tag{1}$$

$$t_{nb} = \frac{\bar{y} + \mathrm{b}(\bar{X} - \bar{x})}{(\bar{x}\rho + \alpha_j)} \left(\bar{X}\rho + \alpha_j\right), b = 7,8, \dots 12, j = 1,2, \dots, 6, \tag{2}$$

$$t_{nb} = \frac{\bar{y} + \mathrm{b}(\bar{X} - \bar{x})}{(\bar{x}\mathrm{Cx} + \alpha_j)} \left(\bar{X}\mathrm{Cx} + \alpha_j\right), b = 13,14, \dots 18, j = 1,2, \dots, 6 \tag{3}$$

The biases and the MSEs of the above estimators are given by,

$$(B(t_{nb}) = \frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{nb}^2, b = 1,2, \dots, 18 \tag{4}$$

$$MSE(t_{nb}) = \frac{(1-f)}{n} (R_{nb}^2 S_x^2 + S_y^2 (1 - \rho^2)) \tag{5}$$

Where,

$$R_{nb} = \frac{\bar{Y}}{(\bar{X} + \alpha_j)}, b = 1,2 \dots, 6, j = 1,2, \dots, 6 \tag{6}$$

$$R_{nb} = \frac{\bar{Y}\rho}{(\bar{X}\rho + \alpha_j)}, b = 7,8 \dots, 12, j = 1,2, \dots, 6 \tag{7}$$

$$R_{nb} = \frac{\bar{Y}\mathrm{Cx}}{(\bar{X}\mathrm{Cx} + \alpha_j)}, b = 13,14 \dots, 18, j = 1,2, \dots, 6 \tag{8}$$

And

$$\alpha_1 = (Md * TM), \alpha_2 = (QD * TM), \alpha_3 = (Md * HL), \alpha_4 = (QD * HL),$$
$$\alpha_5 = (Md * MR), \alpha_6 = (\mathrm{QD} * \mathrm{MR})$$

Yadav, et al., [13] constructed ratio estimators based on both conventional and non-conventional measures that include quartile deviation, decile mean, tri-mean, mid-range, Hodges-Lehmann, Downton's method, Probability weighted moments, Gini's Mean Difference as auxiliary information together with information on the sample size.

$$t_{qe} = \frac{\bar{y} + \mathrm{b}(\bar{X} - \bar{x})}{(\bar{x} + \pi_j)} \left(\bar{X} + \pi_j\right), e = 1,2, \dots 8. j = 1,2, \dots, 8 \tag{9}$$

$$t_{qe} = \frac{\bar{y} + \mathrm{b}(\bar{X} - \bar{x})}{(\bar{x}\rho + \pi_j)} \left(\bar{X}\rho + \pi_j\right), e = 9,10, \dots 16, j = 1,2, \dots, 8 \tag{10}$$

$$t_{qe} = \frac{\bar{y} + \mathrm{b}(\bar{X} - \bar{x})}{(\bar{x}\mathrm{Cx} + \pi_j)} \left(\bar{X}\mathrm{Cx} + \pi_j\right), e = 17,18, \dots 24, j = 1,2, \dots, 8, \tag{11}$$

The biases and the MSEs of the above estimators are given by:-

$$B\left(t_{qe}\right) = \frac{(1-f)}{n} \frac{S_x^2}{\bar{Y}} R_{qe}^2, e = 1,2, \dots, 24 \tag{12}$$

$$(MSE\left(t_{qe}\right) = \frac{(1-f)}{n} (R_{qe}^2 S_x^2 - (1 - \rho^2) S_y^2) \tag{13}$$

Where

$$R_{qe} = \frac{\bar{Y}}{(\bar{X}+\pi_j)}, e = 1,2 \dots ,6, j = 1,2, \dots ,8 \tag{14}$$

$$R_{qe} = \frac{\bar{Y}\rho}{(\bar{X}\rho+\pi_j)}, \; e = 9,10, \dots 16, j = 1,2, \dots ,8 \tag{15}$$

$$R_{qe} = \frac{\bar{Y}\mathrm{Cx}}{(\bar{X}\mathrm{Cx}+\pi_j)}, e = 17,18, \dots 24, j = 1,2, \dots ,8 \tag{16}$$

And

$$\pi_1 = (QD * n), \pi_2 = (DM * n), \pi_3 = (TM * n), \pi_4 = (MR * n),$$

$$\pi_5 = (HL * n), \; \pi_6 = (\mathrm{G} * \mathrm{n}), \; \pi_7 = (\mathrm{D} * \mathrm{n}), \; \pi_8 = (S_{pw} * \mathrm{n})$$

2. Improved Ratio Estimator

Motivated by works of Subzar, et al., [10] and Yadav, et al., [13], the ratio estimator of the population mean is improved utilizing population parameters of an auxiliary variable that are known. This paper proposes a ratio estimator based on quartile deviation, kurtosis coefficient, and non-conventional measure (Tri-mean) and information on the sample size. The suggested ratio estimator is as below.

$$t_{r1} = \frac{\bar{y}+\mathrm{b}(\bar{X}-\bar{x})}{(\bar{x}\beta 2+\chi_1)} (\bar{X}\beta 2 + \chi_1) \tag{17}$$

Where $\chi_1 = \mathrm{QD} * \mathrm{TM} * \mathrm{n}$.

Taylor series method given below in (18) was used to derive the expressions for the bias and the MSE of the suggested estimator.

$$h(\bar{x}, \bar{y}) \cong h(\bar{X}, \bar{Y}) + \frac{\partial(c,d)}{\partial c}|_{\bar{X},\bar{Y}}(\bar{x} - \bar{X}) + \frac{\partial(c,d)}{\partial d}|_{\bar{X},\bar{Y}}(\bar{y} - \bar{Y}) \tag{18}$$

Where, $h(\bar{x}, \bar{y}) = \widehat{R_{ri}}$ and $h(\bar{X}, \bar{Y}) = R$ $with$ $R = \bar{Y}/\bar{X}$

As indicated in Wolter [12] (18) can be applied to the suggested estimator to give expressions of MSE as below:-

For the combination of coefficient of kurtosis, quartile deviation, tri-mean and sample size we have:

$$\widehat{R_{r1}} - R \cong \frac{\partial((\bar{y}+b(\bar{X}-\bar{x})/(\bar{x}\beta 2+\chi_1))}{\partial \bar{x}}|_{\bar{X},\bar{Y}}(\bar{x} - \bar{X}) + \frac{\partial((\bar{y}+b(\bar{X}-\bar{x})/(\bar{x}\beta 2+\chi_1))}{\partial \bar{y}}|_{\bar{X},\bar{Y}}(\bar{y} - \bar{Y}) \tag{19}$$

$$\widehat{R_{r1}} - R \cong -\left\{\frac{\bar{y}}{(\bar{x}\beta 2+\chi_1)^2} + \frac{b((\bar{x}\beta 2+\chi_1)}{(\bar{x}\beta 2+\chi_1)^2}\right\}|_{\bar{X},\bar{Y}}(\bar{x} - \bar{X}) + \frac{1}{(\bar{x}\beta 2+\chi_1)}|_{\bar{X},\bar{Y}}(\bar{y} - \bar{Y}) \tag{20}$$

$$\widehat{R_{r1}} - R \cong -\left(\frac{\bar{y}+ b((\bar{x}\beta 2+\chi_1)}{(\bar{x}\beta 2+\chi_1)}\right)|_{\bar{X},\bar{Y}}(\bar{x} - \bar{X}) + \frac{1}{(\bar{x}\beta 2+\chi_1)}|_{\bar{X},\bar{Y}}(\bar{y} - \bar{Y}) \tag{21}$$

From (21), by squaring on both sides, we have

$$\mathrm{E}(\widehat{R_{r1}} - R)^2 \cong (\frac{(\bar{y}+ B(\bar{x}\beta 2+\chi_1)^2}{(\bar{x}\beta 2+\chi_1)^2})\mathrm{v}(\bar{x}) - 2(\frac{\bar{y}+ B(\bar{x}\beta 2+\chi_1)}{(\bar{x}\beta 2+\chi_1)^3})\mathrm{cov}(\bar{x}, \bar{y}) + \frac{1}{(\bar{x}\beta 2+\chi_1)^2}\mathrm{v}(\bar{y}) \tag{22}$$

$$\mathrm{E}(\widehat{R_{r1}} - R)^2 \cong \frac{1}{(\bar{x}\beta 2+\chi_1)^2}\left[\left(\frac{(\bar{Y}+B(\bar{x}\beta 2+\chi_1))^2}{(\bar{x}\beta 2+\chi_1)^2}\right)V(\bar{x}) - 2\left(\frac{\bar{y}+ B(\bar{x}\beta 2+\chi_1)}{(\bar{x}\beta 2+\chi_1)^3}\right)\mathrm{cov}(\bar{x}, \bar{y}) + \mathrm{v}(\bar{y})\right] \tag{23}$$

Where

$$B = \frac{S_{xy}}{S_x^2} = \frac{\rho S_x S_y}{S_x^2} = \frac{\rho S_y}{S_x}$$

Where $\beta 2$ and χ_1 are the $parameters$ of the $auxiliary$ $variable.$ It should be noted that the difference $[E(b) - B]$ is omitted for it is supposed that the regression line goes through the origin.

Hence the MSE of the proposed estimator that is,

$$MSE(t_{r1}) = (\bar{X}\beta 2 + \chi_1)^2 \mathrm{E}(\widehat{R_{r1}} - R)^2 \tag{24}$$

$$\cong \left[\left(\frac{(\bar{Y}+B(\bar{X}\beta 2+\chi_1))^2}{(\bar{X}\beta 2+\chi_1)^2}\right)V(\bar{x}) - 2\left(\frac{\bar{y}+ B(\bar{X}\beta 2+\chi_1)}{(\bar{X}\beta 2+\chi_1)^3}\right)\mathrm{cov}(\bar{x}, \bar{y}) + \mathrm{v}(\bar{y})\right] \tag{25}$$

$$\cong\left[\left(\frac{(\bar{Y}^2+2B(\bar{x}\beta2+\chi_1)\bar{Y}+B^2(\bar{x}\beta2+\chi_1)^2}{(\bar{X}\beta2+\chi_1)^2}\right)V(\bar{x})-\left(\frac{2\bar{Y}+2B(\bar{X}\beta2+\chi_1)}{(\bar{X}\beta2+\chi_1)}\right)\mathrm{cov}(\bar{x},\bar{y})+\mathrm{v}(\bar{y})\right] \quad (26)$$

$$\cong\frac{1-f}{n}\left[\left(\frac{\bar{Y}^2}{(\bar{X}\beta2+\chi_1)^2}+\frac{2B\bar{Y}}{(\bar{X}\beta2+\chi_1)}+B^2\right)S_x^2-\left(\frac{2\bar{Y}}{(\bar{X}\beta2+\chi_1)}+2B\right)S_{xy}+S_y^2\right] \quad (27)$$

$$\cong\frac{1-f}{n}\left[(R_{r1}^2+2BR_{r1}+B^2)S_x^2-2(R_{r1}+B)S_{xy}+S_y^2\right] \quad (28)$$

$$\cong\frac{1-f}{n}\left[R_{r1}^2S_x^2+2R_{r1}\rho S_xS_y+\rho^2S_y^2-2R_{r1}\rho S_xS_y-2\rho^2S_y^2+S_y^2\right] \quad (29)$$

$$\cong\frac{1-f}{n}\left[R_{r1}^2S_x^2-\rho^2S_y^2+S_y^2\right] \quad (30)$$

From (21), applying the value of B in (28) and evaluating, the MSE of the suggested estimator is obtained as

$$MSE(t_{r1})\cong\frac{1-f}{n}\left[(R_{r1}^2S_x^2+S_y^2(1-\rho^2)\right] \quad (31)$$

Correspondingly the proposed estimator bias is given as

$$Bias(t_{r1})\cong\frac{1-f}{n}\frac{S_x^2}{\bar{y}}R_{r1} \quad (32)$$

3. Efficiency Comparison

Efficiency conditions for the suggested ratio estimator have been derived in relation to the standard ratio estimator and also with the current modified estimators in literature. If the inequality shown below holds, the suggested estimator is more effective than the prevailing estimators.

3.1. Comparison with the Standard Mean Ratio Estimator

The expressions of the MSE of the suggested estimator and the standard mean ratio estimator illustrated below shows the conditions in which the suggested estimator is better than the standard mean ratio estimator.

$$\mathrm{MSE}(t_{r1})\leq\mathrm{MSE}(\widehat{\bar{Y}}_r)$$

$$\frac{1-f}{n}\left[(R_{r1}^2S_x^2+S_y^2(1-\rho^2)\right]\leq\frac{(1-f)}{n}(S_y^2+R^2S_x^2-2R\rho S_xS_y) \quad (33)$$

$$R_{r1}^2S_x^2-\rho^2S_y^2-R^2S_x^2+2R\rho S_xS_y\leq 0$$

$$(\rho S_y-RS_x)^2-R_{r1}^2S_x^2\geq 0,$$

$$(\rho S_y-RS_x+R_{r1}^2)(\rho S_y-RS_x-R_{r1}S_x)\geq 0)$$

Condition 1:

$$(\rho S_y-RS_x+R_{r1}S_x)\leq 0\ and\ (\rho S_y-RS_x-R_{r1}S_x)\leq 0 \quad (34)$$

After evaluating condition 1 we obtain

$$\left(\frac{RS_y-RS_x}{S_x}\right)\leq R_{r1}\leq\left(\frac{RS_x-\rho S_y}{S_x}\right)$$

Which gives

$$\mathrm{MSE}(t_{r1})\leq\mathrm{MSE}(\widehat{\bar{Y}}_r)$$

$$\left(\frac{\rho S_y-RS_x}{S_x}\right)\leq R_{r1}\leq\left(\frac{RS_x-\rho S_y}{S_x}\right)\text{ or }\left(\frac{RS_x-\rho S_y}{S_x}\right)\leq R_{r1}\leq\left(\frac{\rho S_y-RS_x}{S_x}\right),$$

3.2. Comparison with the Estimators in Literature

The expressions of the MSE of the suggested estimator and the current modified ratio estimators illustrated below shows the conditions in which the suggested estimator is better than the estimators in literature.

$$\mathrm{MSE}(t_{r1})\leq\mathrm{MSE}(t_{nb})$$

$$\frac{1-f}{n}\left[\left(R_{r1}^2 S_x^2 + S_y^2(1-\rho^2)\right)\right] \leq \frac{1-f}{n}\left[\left(R_{nb}^2 S_x^2 + S_y^2(1-\rho^2)\right)\right] \quad (35)$$

$$R_{r1}^2 S_x^2 \leq R_{nb}^2 S_x^2$$

$$R_{r1} \leq R_{nb}$$

Where b=1,2,….,18.

Similarly,

$$\text{MSE}(t_{r1}) \leq \text{MSE}(t_{qe})$$

$$R_{r1}^2 S_x^2 \leq R_{qe}^2 S_x^2$$

$$R_{r1} \leq R_{qe}$$

where e=1,2,…,24.

3.3. Percentage Relative Efficiency

The performance of the suggested estimator and current modified estimators in literature are evaluated against the usual mean ratio estimator by computing the percentage relative efficiencies. The highest value of PRE Indicate the most efficient estimator and vice versa. It is computed as follows:-

$$PRE = \frac{MSE\ of\ Mean\ ratio\ estimator}{MSE\ of\ Proposed/Existing\ Estimator} * 100 \quad (36)$$

4. Empirical Study

The performance of the suggested estimator is evaluated and comparison made with the current modified estimators in literature using both simulated and real data. Percentage relative efficiencies are also obtained to evaluate the efficiency of the suggested estimator against the estimators in literature.

4.1. Simulation Study

A simulation study was done in order evaluate the performance of the suggested estimator. R programming was used to generate data from a bivariate normal distribution with different correlation coefficients. A total of 600 simulations were done to obtain data for two populations. Averages for the simulated data was calculated to obtain the following parameters: - population 1: N=1154.5, n=388, ρ= 0.625. N=1155.3, n=388, ρ= 0.91 and population 2: N=1155.3, n=388, ρ= 0.91,. The bias and MSE of the suggested estimator is calculated and compared with that of prevailing estimators.

The results in the tables below indicate that the proposed estimator has low bias compared to some estimators and the least mean squared error hence more efficient than the existing estimators. The PRE of proposed estimator t_{r1} and that of the existing ratio estimators are calculated with respect to the usual mean ratio estimator and the outcomes indicate that the PRE value of t_{r1} was the highest across all three populations implying that the suggested estimator t_{r1} is more efficient than the estimators in literature.

Table 1. *Bias of the existing and suggested estimators for the population mean using simulated data.*

Estimators	Population1	Population2	Estimators	Population1	Population2
$\hat{\bar{Y}}_r$	0.05993	0.04899	t_{n16}	4.3265E-05	2.4982E-05
t_{m1}	0.04899	0.02923	t_{n17}	2.6358E-05	8.0405E-06
t_{m2}	0.04920	0.02547	t_{n18}	3.7133E-05	2.2406E-05
t_{m3}	0.02391	0.02654	t_{q1}	1.6049E-06	2.2628E-06
t_{m4}	0.03872	0.02867	t_{q3}	1.0637E-06	7.6984E-07
t_{m5}	0.04598	0.02407	t_{q4}	9.2087E-07	6.9487E-07
t_{m6}	0.02250	0.02596	t_{q5}	1.0752E-06	7.7576E-07
t_{m7}	0.04191	0.03032	t_{q6}	5.9504E-07	8.3991E-07
t_{m8}	0.04944	0.02557	t_{q7}	7.5725E-07	1.0687E-06
t_{m9}	0.02480	0.02753	t_{q8}	7.5858E-07	1.0705E-06
t_{n1}	2.1611E-05	1.2310E-05	t_{q9}	6.2838E-07	1.8753E-06
t_{n2}	3.0459E-05	3.4213E-05	t_{q11}	4.1631E-07	6.3780E-07
t_{n3}	2.1841E-05	1.2404E-05	t_{q12}	3.6036E-07	5.7567E-07
t_{n4}	3.0783E-05	3.4472E-05	t_{q13}	4.2079E-07	6.4270E-07
t_{n5}	1.8732E-05	1.1120E-05	t_{q14}	2.3277E-07	6.9586E-07
t_{n6}	2.6410E-05	3.0926E-05	t_{q15}	2.9628E-07	8.8544E-07
t_{n7}	8.5153E-06	1.0213E-05	t_{q16}	2.9680E-07	8.8699E-07
t_{n8}	1.2021E-05	2.8419E-05	t_{q17}	2.2646E-06	1.6336E-06
t_{n9}	8.6064E-06	1.0291E-05	t_{q19}	1.5013E-06	5.5546E-07
t_{n10}	1.2150E-05	2.8635E-05	t_{q20}	1.2998E-06	5.0135E-07
t_{n11}	7.3766E-06	9.2242E-06	t_{q21}	1.5175E-06	5.5973E-07
t_{n12}	1.0416E-05	2.5685E-05	t_{q22}	8.4005E-07	6.0604E-07
t_{n13}	3.0399E-05	8.9025E-06	t_{q23}	1.0690E-06	7.7118E-07
t_{n14}	4.2811E-05	2.4794E-05	t_{q24}	1.0708E-06	7.7253E-07
t_{n15}	3.0722E-05	8.9705E-06	t_{r1}	*0.00001*	*0.00001*

Table 2. *MSE of the existing and suggested estimators for the population mean using simulated data.*

Estimators	Population1	Population2	Estimators	Population1	Population2
$\bar{\bar{Y}}_r$	8.30085	2.73139	t_{n16}	6.81777	0.78936
t_{m1}	9.53775	3.56769	t_{n17}	6.81664	0.78775
t_{m2}	10.02874	3.13162	t_{n18}	6.81736	0.78911
t_{m3}	8.42405	3.31135	t_{q1}	6.81497	0.78720
t_{m4}	9.34409	3.42649	t_{q3}	6.81493	0.78705
t_{m5}	9.81883	3.00327	t_{q4}	6.81492	0.78705
t_{m6}	8.28500	3.17738	t_{q5}	6.81493	0.78705
t_{m7}	9.55241	3.57840	t_{q6}	6.81490	0.78706
t_{m8}	10.04455	3.14139	t_{q7}	6.81491	0.78708
t_{m9}	8.43471	3.32154	t_{q8}	6.81491	0.78708
t_{n1}	6.81632	0.78815	t_{q9}	6.81490	0.78716
t_{n2}	6.81691	0.79024	t_{q11}	6.81489	0.78704
t_{n3}	6.81633	0.78816	t_{q12}	6.81489	0.78704
t_{n4}	6.81693	0.79026	t_{q13}	6.81489	0.78704
t_{n5}	6.81612	0.78804	t_{q14}	6.81488	0.78705
t_{n6}	6.81664	0.78992	t_{q15}	6.81488	0.78707
t_{n7}	6.81543	0.78795	t_{q16}	6.81488	0.78707
t_{n8}	6.81567	0.78968	t_{q17}	6.81501	0.78714
t_{n9}	6.81544	0.78796	t_{q19}	6.81496	0.78703
t_{n10}	6.81568	0.78970	t_{q20}	6.81495	0.78703
t_{n11}	6.81536	0.78786	t_{q21}	6.81496	0.78703
t_{n12}	6.81556	0.78942	t_{q22}	6.81492	0.78704
t_{n13}	6.81691	0.78783	t_{q23}	6.81493	0.78705
t_{n14}	6.81774	0.78934	t_{q24}	6.81493	0.78705
t_{n15}	6.81693	0.78783	t_{r1}	*6.75035*	*0.77770*

Table 3. *PRE of the suggested estimator* (t_{r1}) *with the existing estimators using simulated data.*

Estimators	Population1	Population2	Estimators	Population1	Population2
t_{m1}	87.03153	76.55906	t_{n17}	121.7733	346.7331
t_{m2}	82.77062	87.21971	t_{n18}	346.1355	121.7605
t_{m3}	98.53752	82.48569	t_{q1}	346.9754	121.8032
t_{m4}	88.8353	79.71393	t_{q3}	347.0415	121.8039
t_{m5}	84.54011	90.9472	t_{q4}	347.0415	121.8041
t_{m6}	100.1913	85.96359	t_{q5}	347.0415	121.8039
t_{m7}	86.89797	76.32992	t_{q6}	347.0371	121.8044
t_{m8}	82.64034	86.94845	t_{q7}	347.0283	121.8042
t_{m9}	98.41299	82.23264	t_{q8}	347.0283	121.8042
t_{n1}	121.7791	346.5571	t_{q9}	346.993	121.8044
t_{n2}	121.7685	345.6406	t_{q11}	347.0459	121.8046
t_{n3}	121.7789	346.5527	t_{q12}	347.0459	121.8046
t_{n4}	121.7682	345.6318	t_{q13}	347.0459	121.8046
t_{n5}	121.7826	346.6055	t_{q14}	347.0415	121.8048
t_{n6}	121.7733	345.7806	t_{q15}	347.0327	121.8048
t_{n7}	121.795	346.6451	t_{q16}	347.0327	121.8048
t_{n8}	121.7907	345.8857	t_{q17}	347.0018	121.8025
t_{n9}	121.7948	346.6407	t_{q19}	347.0503	121.8034
t_{n10}	121.7905	345.8769	t_{q20}	347.0503	121.8035
t_{n11}	121.7962	346.6847	t_{q21}	347.0503	121.8034
t_{n12}	121.7926	345.9996	t_{q22}	347.0459	121.8041
t_{n13}	121.7685	346.6979	t_{q23}	347.0415	121.8039
t_{n14}	121.7537	346.0347	t_{q24}	347.0415	121.8039
t_{n15}	121.7682	346.6979	t_{r1}	*351.2138*	*122.9692*
t_{n16}	121.7532	346.0259			

4.2. Evaluation on Real Data

Performance of the suggested ratio estimator is evaluated and comparison made with the ratio estimators in literature by use of natural population data from Murthy (1967) page 228 whereby fixed capital is denoted by X (supporting variable) and output of 80 factories shown by Y (main variable). The outcomes in the tables below indicate that the proposed estimator registered the least mean squared. Also, the PRE value of t_{r1} was the highest implying that the suggested estimator t_{r1} is more efficient compared to the prevailing estimators.

Table 4. *Parameters of the natural population under consideration.*

Parameter	Pop 1
N	34
n	20
$\bar{Y}$	856.4117
$\bar{X}$	199.4412
ρ	0.4453
S_y	733.1407
C_y	0.8561
S_x	150.2150
C_x	0.7531
β_2	1.0445
β_1	1.1823
M_d	142.50
TM	89.375
MR	165.562
HL	320
QD	184
G	162.996
D	144.481
S_{pw}	206.944
DM	206.944

Table 5. *Bias of the existing and suggested estimators for the population mean using natural population data.*

Estimators	Bias	Estimators	Bias
$\bar{Y}_r$	4.940	t_{n18}	0.000241
t_{m1}	4.7696	t_{a1}	0.0264
t_{m2}	3.9315	t_{a2}	0.0211
t_{m3}	2.4848	t_{a3}	0.1008
t_{m4}	2.9863	t_{a4}	0.0323
t_{m5}	2.2632	t_{a5}	0.0091
t_{m6}	1.2192	t_{a6}	0.0332
t_{m7}	1.4745	t_{a7}	0.0417
t_{m8}	1.0206	t_{a8}	0.0211
t_{m9}	0.4721	t_{a9}	0.0056
t_{n1}	0.002378	t_{a10}	0.0044
t_{n2}	0.001436	t_{a11}	0.0224
t_{n3}	0.000190	t_{a12}	0.0068
t_{n4}	0.000114	t_{a13}	0.0019
t_{n5}	0.000703	t_{a14}	0.0070
t_{n6}	0.000423	t_{a15}	0.0089
t_{n7}	0.000480	t_{a16}	0.0044
t_{n8}	0.000289	t_{a17}	0.0154
t_{n9}	0.000038	t_{a18}	0.0123
t_{n10}	0.000023	t_{a19}	0.0601
t_{n11}	0.000141	t_{a20}	0.0188
t_{n12}	0.000085	t_{a21}	0.0053
t_{n13}	0.001359	t_{a22}	0.0194
t_{n14}	0.000819	t_{a23}	0.0244
t_{n15}	0.000108	t_{a24}	0.0123
t_{n16}	0.000065	t_{r1}	*0.001411*
t_{n17}	0.000400		

Table 6. *Mean Squared Error of the existing and suggested estimators for the population mean using natural population data.*

Estimators	MSE	Estimators	MSE
$\bar{Y}_r$	10960.76	t_{n18}	8871.97
t_{m1}	12956.54	t_{a1}	8894.403
t_{m2}	12238.71	t_{a2}	8889.867
t_{m3}	10999.75	t_{a3}	8958.069
t_{m4}	11429.27	t_{a4}	8899.409
t_{m5}	10809.96	t_{a5}	8879.587
t_{m6}	9915.939	t_{a6}	8900.235
t_{m7}	10134.57	t_{a7}	8907.471
t_{m8}	9745.846	t_{a8}	8889.867
t_{m9}	9276.033	t_{a9}	8876.52
t_{n1}	8873.8	t_{a10}	8875.544
t_{n2}	8872.993	t_{a11}	8890.955
t_{n3}	8871.926	t_{a12}	8877.608
t_{n4}	8871.861	t_{a13}	8873.368
t_{n5}	8872.365	t_{a14}	8877.788
t_{n6}	8872.126	t_{a15}	8879.38
t_{n7}	8872.174	t_{a16}	8875.544
t_{n8}	8872.011	t_{a17}	8884.936
t_{n9}	8871.796	t_{a18}	8882.268
t_{n10}	8871.783	t_{a19}	8923.232
t_{n11}	8871.884	t_{a20}	8887.892
t_{n12}	8871.836	t_{a21}	8876.267
t_{n13}	8872.927	t_{a22}	8888.381
t_{n14}	8872.465	t_{a23}	8892.677
t_{n15}	8871.856	t_{a24}	8882.268
t_{n16}	8871.819	t_{r1}	*8871.665*
t_{n17}	8872.106		

Table 7. *PRE of the suggested estimator (t_{r1}) and the existing estimators using natural populations.*

Estimators	PRE	Estimators	PRE
t_{m1}	84.59635	t_{n18}	123.5437
t_{m2}	89.55813	t_{a1}	123.2321
t_{m3}	99.64554	t_{a2}	123.295
t_{m4}	95.90079	t_{a3}	122.3563
t_{m5}	101.395	t_{a4}	123.1628
t_{m6}	110.5368	t_{a5}	123.4377
t_{m7}	108.1522	t_{a6}	123.1514
t_{m8}	112.466	t_{a7}	123.0513
t_{m9}	118.1621	t_{a8}	123.295
t_{n1}	123.5182	t_{a9}	123.4804
t_{n2}	123.5295	t_{a10}	123.494
t_{n3}	123.5443	t_{a11}	123.2799
t_{n4}	123.5452	t_{a12}	123.4652
t_{n5}	123.5382	t_{a13}	123.5242
t_{n6}	123.5415	t_{a14}	123.4627
t_{n7}	123.5409	t_{a15}	123.4406
t_{n8}	123.5431	t_{a16}	123.494
t_{n9}	123.5461	t_{a17}	123.3634
t_{n10}	123.5463	t_{a18}	123.4005
t_{n11}	123.5449	t_{a19}	122.834
t_{n12}	123.5456	t_{a20}	123.3224
t_{n13}	123.5304	t_{a21}	123.4839
t_{n14}	123.5368	t_{a22}	123.3156
t_{n15}	123.5453	t_{a23}	123.256
t_{n16}	123.5458	t_{a24}	123.4005
t_{n17}	123.5418	t_{r1}	*123.5479*

5. Conclusion

Use of auxiliary information improves efficiency of ratio estimators. From the study, we have presented a ratio estimator of the population mean by use of auxiliary information of quartile deviation, kurtosis coefficient, Tri-mean and sample size. We have assessed the performance of the suggested estimator both theoretically and in simulation and numerical studies. In all these cases, the proposed estimator performed better than the prevailing estimators. Hence the study concludes that the suggested estimator is more efficient when compared with the existing ones. It is key to note that the population parameters of the auxiliary variable that were used to develop the suggested estimator are robust to outliers. Therefore the proposed estimator may be adopted to obtain more stable results. Additionally, it would be a cost-saving measure if the suggested estimator is applied in practice to efficiently estimate the finite population mean under simple random sampling scheme.

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