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Facrorization of Symmetric and Obliquely Symmetric Polynomials

Rena Eldar Kizi Kerbalayeva[1, 2]

[1]Institute of Mathematics and Mechanics, National Academy Science of Azerbaijan, Baku, Azerbaijan
[2]Middle School № 227, Baku, Azerbaijan

Email address:
rena-kerbalayeva@mail.ru

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Abstract: Meeting some mathematical and algebraic challenges is going to need more mathematical branches to get involved, new ways for mathematical branches interact with other methods and new rules of funding for mathematical paper importantly. The area is so broad that, this paper is not able possibly obtain every problem, but it gives a number of representative examples that the along of this paper that factorizations to be done. I must note that, Algebra is not only a major subject of science, but is also interesting and difficult. This paper is important, not just for Algebra, but for all fields related to mathematics. In addition, factorization of polynomial is one of important and using concept of mathematics. In present paper symmetric and obliquely symmetric polynomials, based on factorization concept have been studied. Furthermore, several integral steps associated with the considered polynomials both of symmetric and obliquely symmetric polynomials type has been recently introduced and in addition factorization of such polynomials have been studied. In this paper I introduce two new and different uses of factorization of symmetric and symmetric polynomials: first we study symmetric polynomials, then we study obliquely symmetric polynomials and we also look through the new idea for factorizations of such type polynomials.

Keywords: Polynomial, Binomial, Trinomials, Factorization, Geometrical Curves

1. Introduction

One can often hear a word “Algebra” and as well as words Jordan Algebras, Chifford Algebras, etc. The first concepts of Algebra have been met in India. Algebraic concepts were given to Europe through Arabia. The paper begins with the central problem of Linear Algebra, which is calling facrorization of symmetric and obliquely symmetric polynomials. The first time we see symmetric polynomials when Albert Girard has been published his book named *New Inventions in Algebra*, where everybody could see a clear definition of elementary symmetric polynomials in 1929. Following Issac Newton has been published his famous work *Newton Identities*, where everybody was able to see concept symmetric polynomials too. This paper’s emphasis on motivation and development, and its availability, make it widely used for algebraic study. Unfortunately there is no such simple method for the factorization of some symmetric or obliquely symmetric polynomials. So we have to introduce one or more possible methods. Factorization of the polynomial is common in science and mathematics. [1, 5, 9, 13]

2. Preliminaries

We shall previously learn the general skill for solving quadratic polynomials. We must note that, polynomial functions can be used to calculate mass of an animal from its traces. In this paper we will develop methods for factorizing and more precisely solving quadratic equations with two variables. Now let us consider following symmetric trinomial

$$ax^2 + bxy + ay^2 \tag{1}$$

and obliquely symmetric trinomial such as

$$ax^2 - bxy + ay^2 \tag{2}$$

where x and y are variables and a and b are constants and they are not equal to zero.

We know, if we substitute x into y and get same polynomial then it calls symmetric polynomial. Similarly, if we substitute –x into y and get same polynomial then it calls obliquely symmetric polynomial.

Throughout the series the trinomial is on understanding Algebra, as well as factorization of trinomials. When some trinomials can be factorized, then factors of this trinomial follow immediately. When it is possible to find factors of given trinomial, then one can solve an equation with two variables respectively. Another way to understand the factorization is with the given new method.

Theorem 1: Let be given polynomials $ax^2 + bxy + ay^2$ and $ax^2 - bxy + ay^2$. These trinomials can be factorized if only and only $b = a_1^2 + a_2^2$. Where a_1 and a_2 are divisors of the number a.

Let us prove it. Suppose that $a = a_1 \times a_2$. Then we hold following

$$ax^2 + bxy + ay^2 = a_1a_2x^2 + a_1^2xy +$$
$$a_2^2xy + a_1a_2y^2 = a_1x(a_1x +$$
$$a_2y) + a_2y(a_2x + a_1y) =$$
$$(a_2x + a_1y)(a_1x + a_2y) \quad (3)$$

and following we yield

$$ax^2 - bxy + ay^2 = a_1a_2x^2 - a_1^2xy -$$
$$a_2^2xy + a_1a_2y^2 = a_1x(a_1x -$$
$$a_2y) - a_2y(a_2x + a_1y) =$$
$$(a_2x + a_1y)(a_1x - a_2y). \quad (4)$$

Once the methods are built, they require to be analyzed so that they are able to introduce needing informations. In order it, let us give some exercises. For example:

Example 1:

Let us consider quadratic trinomial such as $15x^2 + 34xy + 15y^2$. Here a=15 and b=34. Numbers 3 and 5 are divisors of the number 15. In addition $b = 34 = 3^2 + 5^2 = 9 + 25$. Let us complete factorization as far as possible. Hence due to given theorem we have

$$15x^2 + 34xy + 15y^2 =$$
$$15x^2 + 9xy + 25xy + 15y^2 =$$
$$3x(5x + 3y) + 5y(5x + 3y) =$$
$$(5x + 3y)(3x + 5y).$$

Example 2:

Let us consider $8x^2 - 20xy + 8y^2$. Then we can write this polynomial such us

$$8x^2 - 20xy + 8y^2 = 4(2x^2 - 5xy + 2y^2).$$

Here a=2=1$\times$2, b=5=$2^2 + 1^2$. I want to factorize this trinomial. That is why I use new method. With aid of this method I can factorize this trinomial quickly. Then following holds

$$8x^2 - 20xy + 8y^2 =$$
$$4(2x^2 - 5xy + 2y^2) =$$
$$4(2x^2 - 4xy - xy + 2y^2) =$$
$$4\big((x - 2y)2x - y(x - 2y)\big) =$$
$$4(x - 2y)(2x - y).$$

Theorem 2: Similarly, if numbers a and b are co-prime numbers, then polynomials such as $ax^2 + bxy - ay^2$ can factorized if only and only $b = a_1^2 - a_2^2$. Here numbers a_1 and a_2 are divisors of the number a.

Indeed

$$ax^2 + bxy - ay^2 = a_1a_2x^2 +$$
$$a_1^2xy - a_2^2xy - a_1a_2y^2 =$$
$$a_1(a_2x + a_1y) - a_2y \times$$
$$(a_2x + a_1y) = (a_1x - a_2y) \times$$
$$(a_2x + a_1y)$$

and similarly we can write following

$$ax^2 - bxy - ay^2 = a_1a_2x^2 -$$
$$-a_1^2xy + a_2^2xy - a_1a_2y^2 =$$
$$a_1x(a_2x - a_1y) + a_2y \times$$
$$(a_2x - a_1y) = (a_2x - a_1y) \times$$
$$(a_1x + a_2y).$$

[2, 6, 10, 14]

3. Applications

When we factorize some polynomials, we find factors of given polynomial. Finding factors of polynomials we are able to study equations with two variables and we can draw the graph of the curve by potting the two points.

Corollary: Consider polynomials $ax^2 + bxy + ay^2$ and $ax^2 - bxy - ay^2$, where a and b are co-prime numbers, if coefficient b is equal to $b = a_1^2 + a_2^2$ or $b = a_1^2 - a_2^2$ then these polynomials can be factorized. Where numbers a_1 and a_2 are divisors of given coefficient a.

A paper progresses most in mathematics by doing exercises. With practice one may be able to do this method ‘by factorization’. The steps in this would be as follows. That is why let us give some examples. Several examples of this kind will now work out.

Example 3:

Take polynomial $3x^2 + 6xy + 3y^2$. Moreover we can write

$$3x^2 + 6xy + 3y^2 = 3 \times (x^2 + 2xy + y^2).$$

Here $a = 1 = 1 \times 1$ and $b = 2 = 1^2 + 1^2$. Therefore using our method we have

$$3 \times (x^2 + 2xy + y^2) =$$

$$3(x^2 + xy + xy + y^2) =$$

$$3 \times (x + y)(x + y).$$

Indeed it has two factors such as $(x + y)$.

Example 4:

Factorize trinomial $3x^2 + 10xy + 3y^2$. One can easy see that, $a = 3 = 3 \times 1$ and $b = 3^2 + 1^2$. In addition, due to our method, we can write as before

$$3x^2 + 10xy + 3y^2 =$$

$$3x^2 + xy + 9xy + 3y^2 =$$

$$3x(x + 3y) + y(x + 3y) =$$

$$(x + 3y) \times (3x + y).$$

Due to our new method we could factorizate it with simple way.

Example 5:

Consider following triminal such as $6x^2 + 5xy - 6y^2$. Here $a = 6 = 3 \times 2$ and $b = 3^2 - 2^2$. As in the earlier case, we use our new method

$$6x^2 + 5xy - 6y^2 =$$

$$6x^2 + 9xy - 4xy - 6y^2 =$$

$$3x(2x + 3y) - 2y(2x + 3y) =$$

$$(2x + 3y) \times (3x - 2y).$$

Hence taking new method we hold that it has two different binomial factors.

Example 6:

Let us factorize polinomial $12x^2 - 25xy + 12y^2$. Here $a = 12 = 3 \times 4$ and $b = -25 = -(3^2 + 4^2)$. Observe, by using new method, we are able to carry out the factorization of this trinomial:

$$12x^2 - 25xy + 12y^2 =$$

$$12x^2 - 9xy - 16xy - 12y^2 =$$

$$3x(4x - 3y) - 4y(4x - 3y) =$$

$$(4x - 3y) \times (3x - 4y).$$

The following solving examples illustrate how to use these methods for factorizations. What we do is to observe the given polynomials. [4, 7, 11]

4. Main Results

After polinomials, curves are probably the most familar concept in all mathematics. Analytic Geometry is not studied very much. But curves, ellipses, parabolas and hyperbolas are of great importance in both pure and applied mathematics. One of execellent examle of a typical problem of geometry is study curves. In this paper I give an alternative method of teaching geometric curves, which would develope behaviors of some curves. Curves used by scheting in the plane and I must note that their applications are many important and different. I propose to note that, approaching algebraic conceptes and geometric concepts is very important for mathematics. Let us assume that, we have taken the algebraic approach to geometric approach. What can we get? In the Analytic Geometry we learn various type curves, which the set of all their points belong to the plane. In Analytic Geometry we are concerned chiefly with equations in two or more variables. Every curves satisfies some equations. We know that, curves may determine with equation in general $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. Taking $D = E = F = 0$ we can hold given form of trinomials. One can show that, homogeneous quadratics $ax^2 + bxy + cy^2$ and $ax^2 - bxy + ay^2$ may be determine some curve. These equations enable us to understand why factorization of polynomials is very important for mathematics. [16]

5. Conclution

In branches, where important development had taken place, such as widely interest of many new methods published: matrices, linear equations, vector spaces, subspaces, linear transformations, determinats, polynomials, groups and rings, etc. Some of applications of Algebra include to use of quadratic trinomials in our living. Factorization is one of most using concept of mathematics. We know that, expressions that contain exactly one, two and three terms are called monomials, binomials and trinomials respectively. When somebody factorizes an algebraic expression, then one can write it as a product of factors. It is very interesting, but it is also difficult. In many real problems, trinomials are factorized for which factors using analytic or non-traditional methods are not possible, but for which you nonetheless want to know factors. Algebra needs to developed systematic methods to factorize these quadratic trinomials, i.e., to find their needing factors. In this paper I introduced algebraic method for factorization of such trinomials. One can see how, for many problems on factorization of trinomial expressions, use of given meyhod gives a simple alternative method of solving them. I thing that for factorization of quadratic trinomial is quicker than the knowing simple method. I did so by considerin some examples. I have been gaven six examples, which one could see below. Even though, I have worked out only a few examples, one can see that any trinomial, which coefficents of this trinomial satysfing gaven condition, can be factorized by such easy method. In addition, to apply it, first somebody will see the geometrical representations of inear and quadratic polynomials and the geometrical meaning of their zeroes. [3, 8, 12, 15]

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